ED–2871

B. C. A. (Part III) EXAMINATION, 2021

Paper First

CALCULUS AND GEOMETRY

Time : Three Hours

Maximum Marks : 50

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) State and prove fundamental theorem of integral calculus.

   (b) Let $f(x) = x^3$ on [0, 2], $2 > 0$. Show that if

   $f \in R$ [0, 2] and $\int_0^2 x^3 dx = \dfrac{2^4}{4}$

   (c) Prove that if f is montonge on [a, b] and let 2 be continuous and increasing on [a, b]. Then $f \in R$ (2)

Unit—II

2. (a) Discuss the maximum values of u in

   $$u = xy + \frac{a^3}{x} + \frac{a^3}{y}$$

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(b) Find the maximum value of $x^2 + y^2 + z^2$ having given :

$$ax + by + cz = p$$

(c) Find maximum and minimum value of $u = \sin x \, \sin y \, (x + y)$

Unit—III

3. (a) Show that $\int_1^\infty \sin x^2 dx$ is convergent.

(b) Test for convergence of $\int_0^1 x^{n-1} \log x \, dx$.

(c) Test for convergence of $\int_0^1 x^{n-1} e^{-x} \, dx$ if $o < n < 1$

Unit—IV

4. (a) Find the equation of the came whose vertex is (1, 2, 3) and base curve is the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$

(b) Find the angle between the lines of section of the plane $3x + y + 5z = 0$ and the came $6yz - 2zx + 5xy = 0$

(c) Find the equation of the right circular cylinder whose radius is 2 and axis is the line

$$\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{}$$

Unit—V

5. (a) If PSP' is the focal chord of a conic $\dfrac{l}{r} = 1 + ec \to o$ whose focus is S, then show that :

$$\frac{1}{SP} + \frac{1}{SP1} = \frac{2}{l}$$

(b) Find the equation of the normal at a point ' α ' on the conic.

$$\frac{l}{r} = 1 + e \cos \theta$$

(c) If the tangoues at any point of an ellipse makes an angle α with its major axis and an angle β with the focal radius to the point of contact, then show that.

$$e \cos \alpha = \cos \beta$$