ED-621

M.A./M.Sc. 3rd Semester
Examination, March-April 2021

MATHEMATICS

Optional (B)

Paper - V

Graph Theory - I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) Prove that if a graph H is Homeomorphic from a graph G, then G is a contraction of H.

(*b*) Prove that if G is a k-regular graph, k is an eigenvalue of G. Then this is simple if G is connected. Every other eigenvalue has absolute value $\leq k$.

(*c*) Define the following terms with an example :

(*i*) Direct sum

(*i*) Direct product

(*iii*) Derived graph

(*iv*) Vector space

2. (*a*) Prove that the incidence matrix F of a graph G has rank n-k, where k is the number of components.

(*b*) Prove that any square submatrix of the adjacency matrix F of a graph G has determinant $+1$, -1 or zero.

(*c*) Prove that the sum of any two cuts of a graph G is also a cut of G.

3. (*a*) Prove that if G is a critical graph, then $\delta(G) \geq k-1$.

(*b*) Prove that any uniquely k-colourable graph is $(k-1)$ connected.

(*c*) Prove that for any graph G with order $n \geq 4$, $l(G) \leq [n^2/4]$.

4. (*a*) Prove that for any graph G, $\alpha_0 + \beta_0 = n$.

(*b*) Prove that for any graph G, $c(G) = p(G)$.

(*c*) Prove that for any graph G of order $n \geq 2$ without isolated vertices, $\pi_i \leq [n^2/4]$ and the partition need use only edge and triangles.

5. (*a*) Prove that a graph is triangulated iff every mimimal vertex separator induces a complete subgraph.

(*b*) Prove that every comparability graph is perfect.

(*c*) Prove that a graph G is a permutation graph iff G and $\overline{G}$ are comparability graphs.